Notes on Neugebauer ACT
Lunar theory in Old Babylon
Seminar on Astronomy in Old Orient

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Contents

1 Introduction

These are notes made by myself during studying of the ACT book by Neugebauer. As I found many of the mathematical descriptions cumbersome and sometimes misleading, I have tried to give the calculation recipes a modern style, adding such terms as *velocity* and *angle*, thoroughly omitted by the author, and being clear about units in those rare cases in which the author is not. There seem to be no doubt about the existence of a System A distinct from System B; however I personally feel a dislike towards such a distinction, as the richness of approximations done by Oriental astronomers to what we call today trigonometric functions goes far beyond a simple distinction between a piecewise linear and a piecewise constant function, the basis of the separation between System A and B. I will not emphasize a general difference between the systems, but list those calculations, which indeed do differ in parallel so as to be compared.

These notes involve those chapters which deal with the lunar theory for calendaric purposes. The eclipses are not very much an issue here. Also these notes do not treat some very involved calculations which I found too technical to be presented in full detail. Also calculation methods we do not know much about are not presented.

I have permitted myself to rename the columns and values. Each renaming is stated in the text on first mentioning and is listed in the glossary.

2 Modern view on Babylonian calculations

The civil calendar of the Babylonians was dividing the year into months and months into days. The month corresponds to what today is called a **synodic lunar period**, being the time difference between two new moons. As you in practice will not be able to recognize a new moon as such all the day and even not all the night, necessarily this difference has to be rounded up to full days. A day is defined by the motion of the Earth just as we are used to, it is however starting and ending at sunset. In order to keep a synchronisation with the solar year, the one important for agriculture, leap months had been introduced; we will not discuss those here, as the fact of their introduction is sufficient, the precise moment being somewhat arbitrary.

The aim of a calendar is to predict the exact day at which the new crescent will be visible above the horizon. This day will then be called first day of a new month. In other words the aim of the astronomical calculations for the calendar has been to predict whether the next month will 29 or 30 days. The effects unknowingly taken into account by the Babylonians are quite impressive. We are not led to assume that the calculation algorithms they used were somehow connected to a model of the world. It seems to be a collection of empricial laws. It would thus be nonsensible to say, that the Babylonians did know that the orbit of the Earth is not a circle. But I would like to put it like this: if you would have told them, that it were a circle, they would have disagreed on the basis of what they knew from empirical observations!

If the Earth around the Sun and the Moon around the Earth would both move on circles whose planes coincide, then all calendaric calculations would have been exceedingly simple. In fact at each conjunction the new moon would perform an eclipse, and the time difference between those events would be constant throughout the year. The reason for why we do not have an eclipse every month is that the plane of the moon is slightly inclined[1] with respect to the orbital plane of the Earth. Thus the Moon is often slightly off to be really in front of the Sun. A conjunction is then the moment when both share the same right ascension on the sky (are closest), however definitions can vary here. If one calculates to lowest order the effect of the tilt one finds a sinusoidal deviation of the position of the moon from the position of the sun, they can match only twice a year. These positions lie on what is called the nodal line, namely the line shared by the two orbital planes in question. If the Moon does by chance fnd itself in one of these points at the right time of the year, when they are directed towards the Sun as seen from the Earth, a solar eclipse takes place. The Old Babylonians knew this effect and modeled it in their calculations.

[1]It is here to be emphasized that by laws of mechanics (conservation of angular momentum) this plane cannot rotate or tilt in space, but is carried along with the movement of the Earth!

Neither the orbit of the Earth nor that of the Sun is a circle. These deviations are measurable and have to be taken into account, they are however still small enough to be treated what is called perturbatively. In general one would have to find a self-consistent solution if one wants to know the position of the Moon relative to the Sun. The Old Babylonians calculated this approximatively by taking an exact solution for e.g. a circular orbit of the Moon and some annual velocity law of the Earth around the Sun (they thought it the other way round, of course) and applying corrections due to the ellipticity of the orbit of the Moon. All the deviations are (to first order) again of sinusoidal form.

The last thing one needs to account for in order to perform a prediction of the new moon is the varying length of day over the year. This is a purely geometric effect, where taking into account the ellipticity of the orbit or the shape of the Earth would yield what is called second-order-corrections, negligible in this approach. The resulting function can be approximated for typical latitudes of the region of Babylon again by a sinusoidal one. It is remarkable how the piecewise linear approximation found in procedure texts matches the real curve. It must also have been by far the most precisely measured curve. The measurements of the solar motion are by far more involved, and could have been inferred possibly only by their introduction into the calculations and retrospective validation of the results. This would explain the very crude approximation by e.g. a piecewise constant function. The Babylonian function of daylight for any time of the year shows that they could have done much better.

3 Position of the Sun

The position of the moon during a new one is that of the Sun, corrected for the inclination of the orbit. The speed of the Sun has been assumed to be a piecewise constant function in System A, and a piecewise linear function in System B, this being the defining distinction between the two.

In System A the speed $v_{\odot}$ is set to

$$30\,\text{deg} \;/\; 30\text{ days}$$

for the quick arc between ♍13 and ♓27 and to

$$28;7,30\,\text{deg} \;/\; 30\text{ days}$$

on the slow arc for the rest of the year, equal to $28\frac{1}{8}$ which is $\frac{15}{16}$ of the speed on the quick arc. The solar apogee being in ♊ explains the position of the slow arc.

In System B the speed is a piecewise linear function consisting of straight lines joining the maximum of

$$30;1,59,0\,\text{deg} \;/\; 30\text{ days}$$

in ♐ and the minimum of

$$28;10,3940\,\text{deg} \;/\; 30\text{ days}$$

in ♊ (unabbreviated parameters, ACT 1 p. 70) with a monthly increase of 18,0,0.

3.1 System A

What is noted in the corresponding column gives the position d of the Sun at time distances of 30 days, apparently assumed to be the same as a month for reasons I do not know. As the zodiacal signs are cut into 30 degrees, this is a very simple calculation for the quick arc, and a fairly simple one for the slow arc. Let us consider as an example the plate No. 9 obv. III:

	[še]	16,18,45	hun	♈	$+28,7,30-30$	
3,5	[bar]	14,26,15	múl	♉	$+28,7,30-30$	
	[gu$_4$]	12,33,45	maš	♊	$+28,7,30-30$	
	[sig]	10,41,15	kušu	♋	$+28,7,30-30$	
	[šu]	8,48,45	a	♌	$+28,7,30-30$	
	[izi]	6,56,15	absin	♍	$+29,35,45-30$	$\Delta d^{\text{slow}} = 6;3,45$ $\Delta d = 30 - 0;24,15$
	kin	[6,32]	rín	♎	$+30-30$	
	dy	[6,32]	gír-tab	♏	$+30-30$	
	apin	6,[32	p]a	♐	$+30-30$	
	gan	6,32	[maš]	♑	$+30-30$	
	ab	6,32	g[u]	♒	$+30-30$	
	[zíz]	6,32	zib-me	♓	$+29,24,15-30$	$\Delta d^{\text{quick}} = 20;28$ $\Delta d = 28;7,30 + 1;16,45$
	[še]	5,56,15	[hu]n	♈		

What has to be done in System A when one passes over a discontinuity, Δd^{slow} or Δd^{quick} being the position difference between the last value given and a discontinuity [times in days], is:

$$30 = \frac{\Delta d^{\text{slow}}}{\frac{v^{\text{slow}}}{30}} + \frac{\Delta d^{\text{quick}}}{\frac{v^{\text{quick}}}{30}} \tag{1}$$

$$1 = \frac{\Delta d^{\text{slow}}}{v^{\text{slow}}} + \frac{\Delta d^{\text{quick}}}{v^{\text{quick}}} \tag{2}$$

$$\Delta d^{\text{quick}} = v^{\text{quick}} - \Delta d^{\text{slow}} \frac{v^{\text{quick}}}{v^{\text{slow}}} \quad \text{or} \quad \Delta d^{\text{slow}} = v^{\text{slow}} - \Delta d^{\text{quick}} \frac{v^{\text{slow}}}{v^{\text{quick}}} \tag{3}$$

$$\Delta d := \Delta d^{\text{slow}} + \Delta d^{\text{quick}} = v^{\text{quick}} + \Delta d^{\text{slow}} \left(1 - \frac{v^{\text{quick}}}{v^{\text{slow}}}\right) \text{ or } v^{\text{slow}} + \Delta d^{\text{quick}} \left(1 - \frac{v^{\text{slow}}}{v^{\text{quick}}}\right) \tag{4}$$

$$\Delta d = 30 - \Delta d^{\text{slow}} \cdot \frac{1}{15} \quad \text{or} \quad 28;7,30 + \Delta d^{\text{quick}} \cdot \frac{1}{16} \tag{5}$$

$$= \Delta d^{\text{slow}} + \frac{16}{15}(28;7,30 - \Delta d^{\text{slow}}) \quad \text{or} \quad \Delta d^{\text{quick}} + \frac{15}{16}(30 - \Delta d^{\text{quick}}) \tag{6}$$

These are the formulae stated in ACT I p. 46 (5a), and more important those found in procedure text 200. In translation from ACT I:

en 27 absin$_0$..........	From ♓13 to ♍27 month by month (you shall add)
28,7,30 *šá al* 13 z[ib]	28;7,30, anything beyond ♓13
dirig a-rá 1,4 DU ki	multiply by 1,4 (and) add
13 zib tab ta 27 absin	it to ♓13. From ♍27
en 13 zib 30 tab *ša al*	to ♓13 you shall add 30, anything beyond
27 absin$_0$ dirig a-rá 56,15	♍27 multiply by 56,15
DU ki 27 absin tab	(and) add it to ♍27

The values occuring are $1;4 = \frac{16}{15}$ und $0;56,15 = \frac{16}{15}$. However this column refers to what is called B_2 in ACT I, a modified column, referring not to new, but to full moons and having inversed arcs with slightly different jumping points.

3.2 System B

Let us consider plate No. 122 Obv I and II:

	[XII]	[29,8,3]9,18	$+42,0,0$	2,2,6,20	hun	♈	$+28,7,30-30$
[3,28	I	[28,50,39,]18	$-18,0,0$	[5]2,45,38	múl	♉	$+28,7,30-30$
	[II]	[28,3]2,39,18	$-18,0,0$	29,25,24,56	múl	♉	$+28,7,30-30$
	[III]	[28],14,39,18		27,40,4,14	maš	♊	$+28,7,30-30$
	[IV]	[2]8,24,40,2	$+18,0,0$	26,4,44,16	kušu	♋	$+28,7,30-30$
	[V]	[2]8,42,40,2	$+18,0,0$	24,47,24,18	a	♌	$+29,35,45-30$
	[VI]	29,·,40,2	$+18,0,0$	23,48,4,20	absin	♍	$+30-30$
	[VI$_2$]	29,18,40,2	$+18,0,0$	23,6,44,22	rín	♎	$+30-30$
	[VII]	[2]9,36,40,2	$+18,0,0$	22,43,24,24	gír-tab	♏	$+30-30$
	[VII]	29,54,40,2		[22,38],4,26	pa	♐	$+30-30$
	[IX]	[29],51,17,5[8]	$-18,0,0$	[22,29,22],24	maš	♑	$+30-30$
	[X]	[29],33,17,58	$-18,0,0$	[22,2,40,22	g]u	♒	$+29,24,15-30$
	[XI]	[2]9,15,17,58		21,17,58,20	zib-me	♓	

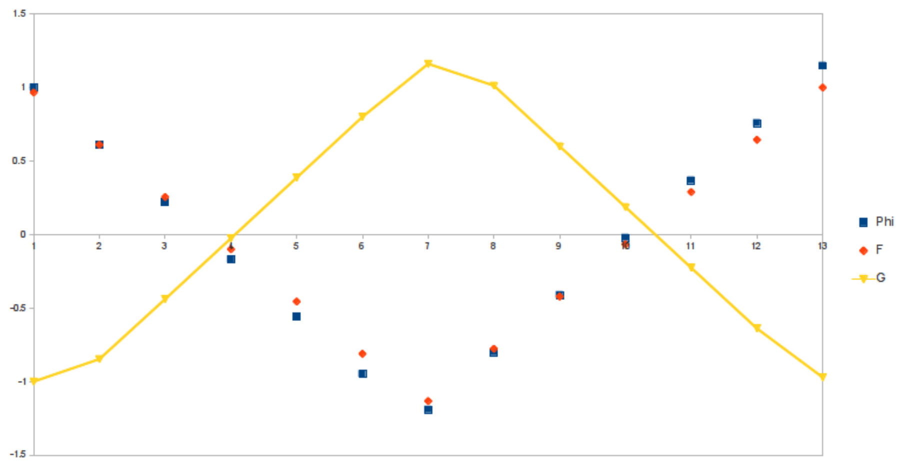
The first column gives the velocity of the Sun, measured in degrees per month, and the second is adding them up in order to get the positions as a function of time. We do not discuss the methods of calculation when one passes over the maximum or minimum, but they are straightforward.

4 Velocity of the moon and length of month

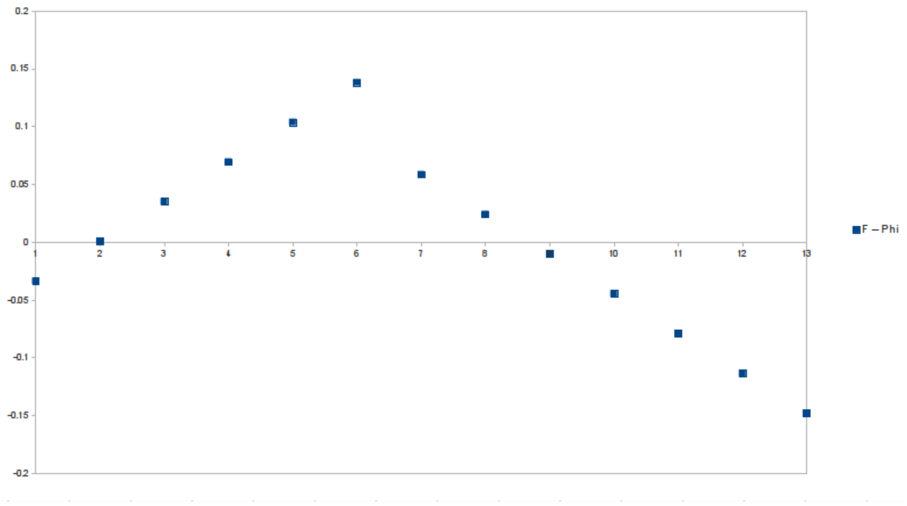
What Neugebauer calls Columns Φ and F is mentionned in the procedure text 200 Sec.5 as “procedure for the velocity of the moon”.

	Column $\hat{\mathrm{G}}$	**Column F**	**Column** Φ
d	25,48,38,31,6,40	42,0,0,0	2,45,55,33,20
M	5,4,57,2,13,20,0	15,56,54,22,30	2,17,4,48,53,20
m	2,4,59,45,11,6,40	11,4,4,41,15	1,57,47,57,46,40
p	0;55,59,6,...	0;55,59,6,...	0;55,59,6,...

As a first approximation only the variable speed of the Moon is taken into account. The values given are diminished by the obvious part of 29^{d}. For a synodic month the moon has to accomplish more than just one (sidereal) period. Let us assume just for the sake of simplicity not the real orbit of an ellipse with the velocity law being a rather complicated function of its parameters, but just a circular orbit with time-dependent velocity, which is unphysical as it violates angular momentum conservation, but is nice as we have a better intuition for what happens in such a system. The length of the sidereal period is not affected by the changing velocity, as the moon time interval per period during which the moon is, say, on the quick arc, is the same throughout the year. However for the synodical period, defining a month, the moon has to travel a slight extra arc. Now it depends whether this extra bit happens to lie on the quick, or the slow arc. This changes throughout the year. Thus actually there is indeed a causal dependence between the column of lunar velocities (called F) and the column for the length of the month (called G). However Neugebauer states that there is just one procedure text (208) which does indeed connect the two. The calculations seem to use column Φ instead, whose meaning is still not clear, although it is numerically closely related to the lunar velocity.



Plotting the three columns in question from Plate No. 9 reveals three graphs which, normalised to unity amplitude and shifted to zero mean value produce the above figure. With this changes it is easy to find that the difference between Φ and F is just again a piecewise linear function.



Whereas the velocity of the moon has been assumed a piecewise linear function, the length of the month has been slightly adjusted around the maximum adn minimum points vie a tabulated interpolation procedure. Neugebauer works with a proto-G function called $\hat{G}$. It oscillates between 2,4,59,... and 5,4,57,... (citing the first three digits of the real value for brevity) and conincides with G for values in [2,53,20,...; 4,46,42,...]. G itself reaches the extremal values of2,40,0,... and 4,56,35,... .

The values in System B are similar, although the precise number and content of the involved columns is not the same.

5 Latitude of the moon

The Moon during the conjunction is not sharing the Sun's position, but is slightly off due to the inclination of its orbital plane relative to that of the Earth. The units used here are interpreted by Neugebauer to be "barleycorns" (še), $1^{\text{še}}$ being 0;0,50°.

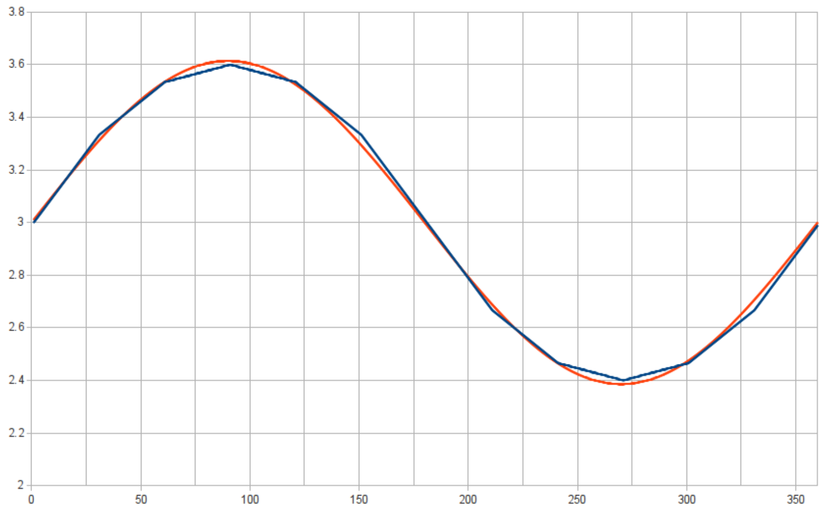
The calculation starts with the lateral velocity of the moon described by a step function between the values 1,58,45,42 and 2,6,15,42 with the same jumping points as the solar velocity of System A. This is at first sight puzzling, because these jumping points for the velocity of the Sun are dictated by the orientation of the Earth's ellipse relative to the stars, for the lateral velocity they are given by the orientation of the common line between the planes of the two orbits in question (nodal line), and a priori these two do not need to have anything in common. I have not checked, what the orientation of the Moon's plane really is, and whether the velocity maxima do indeed (by chance) correspond.

The actual position of the moon is now given by a summation of the path for equidistant time intervals, as for the sun. However modifications arise here. First the extremal values allowed for the lateral positions are ±7,12,0,0. Thus starting at some point and adding each time a value corresponding to the slow arc velocity yields a linear function of some slope. Upon arrival at the extremal value the incremental direction is reversed. One would end up with an asymmetric piecewise linear function. However even more complications are introduced. Inside the so-called nodal zone lying between the values ±2,24,0,0 the increment is doubled. Thus the curve passes with double the slope through this narrow strip. Of course one normally has to account for the fact that the actual passage through its boundary occurs between two values, and one has to calculate first the slow part before entering the nodal zone, and then the quick part, and adding them up obtain an increment for the complete month.

This column presents an example of a very involved calculation method, and we omit here a detailed derivation of all the rules.

6 Length of daylight

The length of daylight T is given in a tabular form in procedure text No. 200 Sec. 2 for the 10^{th} degree of each zodiac (starting with the tenth is explained by the equinox being at 10° ♈), with a linear interpolation rule inbetween. In the following I have plotted the theoretical curve for the length of daylight at approximately the latitude of Babylon over the curve one would obtain when using their table. The Babylonian interpolation of the sine is of amazing precision.



A similar curve has been reconstructed in System B, it contains only slightly altered values and is furthermore nowhere known to have been used exactly, but we are probably presented with rounded-off values.

7 Determination of the length of month

The aim of the claendaric calculations has been to determine whether a given month will have 30 days (full) or 29 (hollow). Given the moment of the last conjunction and the time interval to the next. This interval is what is called in ACT I Column G, enlarged by

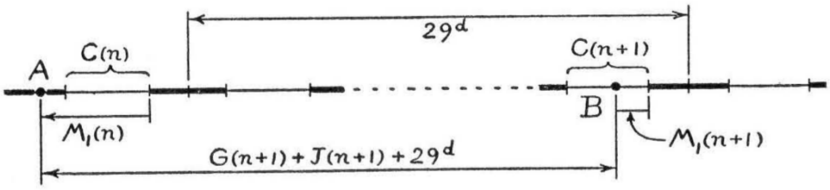


Fig. 27

the correction from Column J. This corresponds to a calculation of the moments of the conjunction first under the assumption of a constant solar velocity, and afterwards taking into account a small correction due to its actual nonconstancy.

Given the lengths of daylight for the two months in question we calculate:

time from last conjunction to subsequent sunset + half of night last month
+29 days
−half of night new month − time from new conjunction to subsequnt sunset
=time between two conjunctions

Here we know that the synodic month is between 29 and 30 days long, thus between the midnights following the conjunctions lie exactly 29 days. If all days were equally long the fomula would be simply

time from last conjunction to subsequent sunset
−time from new conjunction to subsequnt sunset
=time between two conjunctions − 29 days

A particularity of these calculations in System B is the explicit use of the midnight epoch, changing the date at midnight and not at sunset, which evidently has computational advantages.

8 Actual visibility of the new crescent

We will not state here the rules applied for the determination of the actual visibility conditions of the new crescent for several reasons. First they actually are not of astronomical, but more meteorological nature, knowledge which of course cannot have been present to the Babylonians. Second, the rules are not really well known. This is not at all astonishing, as this determination is a very complicated procedure and with all the mathematical and statistical tools available those days it cannot have been possible to predict this accurately, just for the simple reason of randomness. Thus we cannot expect any real *rules* going beyond a rule of thumb. And these latter ones however are hard to infer. Thus we only state here their presence, but refer the interested reader to ACT book; he should however be aware of the above-mentionned difficulties.

9 Glossary

Name in ACT	**Name here**
Column A	$v_\odot$
s	t^{slow}
σ	Δd^{quick}
s	Δd^{slow}
σ_1 or s_1	Δd
Column C	T